

INTERACTING QUINTESSENCE AND THE COINCIDENCE PROBLEM

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We investigate the role of a possible coupling of dark matter and dark energy. In particular, we explore the consequences of such an interaction for the coincidence problem, i.e., for the question, why the energy densities of dark matter and dark energy are of the same order just at the present epoch. We demonstrate, that, with the help of a suitable coupling, it is possible to reproduce any scaling solution $\rho_X \propto \rho_M a^\xi$, where a is the scale factor of the Robertson-Walker metric and ξ is a constant parameter. ρ_X and ρ_M are the densities of dark energy and dark matter, respectively. Furthermore, we show that an interaction between dark matter and dark energy can drive the transition from an early matter dominated era to a phase of accelerated expansion with a stable, stationary ratio of the energy densities of both components.

1. Introduction

Our Universe seems to be made of roughly two third of something which is now called dark energy (DE) and of about one third of mainly cold dark matter (CDM). Usually, these components are considered as independent ingredients of the cosmic substratum. Given, that we neither know the nature of DE nor that of CDM, one cannot exclude that there exists a coupling between them. Such a situation is even the more general one compared with the uncoupled case. Here we explore some of the consequences that different types of interactions may have for the dynamics of the Universe.

2. Scaling solutions

A phenomenological generalization of the Λ CDM model, characterized by $\rho_M \propto a^{-3}$ and $\rho_X \equiv \rho_\Lambda = \text{const}$, is¹

$$\frac{\rho_M}{\rho_X} = r_0 \left(\frac{a_0}{a} \right)^\xi, \quad (1)$$

with a dark energy equation of state parameter $w_X \equiv p_X/\rho_X$ which is assumed to be a negative constant. For $\xi = 3$ one obviously recovers the Λ CDM model. The limit $\xi = 0$ corresponds to stationary ratio of the energy densities. For any value $\xi < 3$ the coincidence problem is less severe than for the Λ CDM model. Here we want to show, that $\xi < 3$ can be achieved by a suitable interaction according to

$$\dot{\rho}_M + 3H\rho_M = Q, \quad \dot{\rho}_X + 3H(1 + w_X)\rho_X = -Q. \quad (2)$$

Scaling solutions of the type (1) are then obtained for a choice

$$Q = -3H \frac{\frac{\xi}{3} + w_X}{1 + r_0(1+z)^\xi} \rho_M. \quad (3)$$

This demonstrates that by a suitable interaction one may reproduce any desired scaling behavior of the energy densities.² The uncoupled case is recovered for $\xi + 3w_X = 0$. For any $\xi + 3w_X \neq 0$ we obtain an alternative cosmological model.

2.1. The case $w_X = -1$, $\xi = 1$

In such a case we have $Q > 0$, i.e., a continuous energy transfer from the X -component to the matter fluid and a total energy density

$$\rho = \frac{\rho_0}{(1+r_0)^3} \left[1 + r_0 \frac{a_0}{a} \right]^3. \quad (4)$$

Compared with the corresponding expression of the Λ CDM model, $\rho_{(\Lambda\text{CDM})} \propto \rho_0 \left[1 + r_0 (a_0/a)^3 \right]$, the sum of the powers (Λ CDM) is replaced by the power of a sum. For the luminosity distance of this model we obtain an expression which differs from the corresponding one of the Λ CDM model only in the third order in the redshift. While the results of both models are indistinguishable from the current SN Ia data, the coincidence problem is less severe in the interacting model.²

2.2. The stationary case $\xi = 0$

This case has $\rho_X, \rho_M \propto a^{-\nu}$, where $\nu = 3(1 + r_0 + w_X)/(1 + r_0)$ and $a \propto t^{2/\nu}$. There is accelerated expansion for $\nu < 2$, corresponding to $3w_X < -(1 + r_0)$. Modelling the X component by a scalar field, the corresponding potential is

$$V(\phi) \propto \exp[-\kappa\phi], \quad \kappa = \sqrt{\frac{24\pi G}{(1 + w_X)(1 + r_0)}} [1 + w_X + r_0]. \quad (5)$$

The condition for accelerated expansion translates into $\kappa^2 < 24\pi G \frac{w_X^2}{(1+r_0)(1+w_X)}$. We conclude that there exists a specific interaction which admits a stationary ratio of the energy densities and is compatible with an accelerated expansion of the Universe.³ In a next step we outline, how such a state might be dynamically approached.

3. Asymptotically stable solution

With an ansatz $Q = 3Hc^2\rho$ for the interaction, equations (2) admit two stationary solutions, r_s^\pm , for the ratio $r \equiv \rho_M/\rho_X$ of the energy densities. These have the properties $r_s^+ - r_s^- \geq 0$ and $r_s^+ r_s^- = 1$. For $r = r_s^+$ we have $\rho_M > \rho_X$, i.e., matter dominance, while for $r = r_s^-$ the reverse relation $\rho_M < \rho_X$ holds, equivalent to a dark energy dominated state. The dynamics of r in terms of these stationary solutions is given by

$$\dot{r} = -3c^2 H [(r - r_s^-)(r_s^+ - r)] . \quad (6)$$

While r_s^+ can be shown to be unstable, r_s^- is a stable, stationary state. There exists a solution

$$r(y) = r_s^- \frac{1 + yr_s^+}{1 + yr_s^-} , \quad r(1) = 1 , \quad (7)$$

with $y = (a_{eq}/a)^\lambda$ and $\lambda = -3w_X(1 - r_s^-)/(1 + r_s^-)$, according to which the density ratio evolves from the unstable stationary value r_s^+ for $y \gg 1$ (matter dominance) to the stable, stationary state r_s^- for $y \ll 1$ (dark energy dominance).⁴ The deceleration parameter decreases with the expansion. In particular, it may switch from positive to negative values. Assuming as an example $w_X = -1$, $r_0 \approx 3/7$ and, say, $r_s^- \approx 1/7$, we find that the value r_{acc} of r at which this transition occurs is $r_{acc} \approx 2$. This corresponds to a redshift of $z_{acc} \approx 1.6$.

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